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6 SEM TDC DSE MTH (CBCS) 1 (H)

2023

(May/June)

MATHEMATICS

(Discipline Specific Elective)

(For Honours)

Paper : DSE–1

(Hydromechanics)

Full Marks : 80
Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* State two methods of studying fluid motion mathematically. 1

(b) Choose the correct answer : 1

Velocity potential ϕ satisfies which of the following equations?

(i) Bernoulli

(ii) Cauchy

(iii) Laplace

(iv) None of the above

(c) Define velocity potential. Under what condition, the flow is known as the potential kind? 2+1=3

(d) The velocity components in three-dimensional flow for an incompressible fluid are $(2x, -y, -z)$. Is it a possible field? Determine the equation of streamline passing through (1, 1, 1). 2+3=5

(e) Deduce the equation of continuity by Euler's method. 5

Or

Express the acceleration of a fluid particle in Cartesian coordinate.

2. (a) Write the Euler's equation of motion. 1

(b) State True or False : 1

Impulsive pressure at any point in a fluid is the same in every direction.

(c) If the motion of an ideal fluid, for which density is a function of pressure p only is steady and the external forces are conservative, then prove that there exists a family of surfaces which contains the streamlines and vortex lines. 4

(d) Derive Bernoulli's equation of motion of fluid. 6

Or

A stream is rushing from a boiler through a conical pipe, the diameters of the ends of which are D and d. If V and v be the corresponding velocities of the stream and if the motion be supposed to be that of the divergence from the vertex of the cone, then prove that

$$\frac{v}{V} = \frac{D^2}{d^2} e^{(v^2 - V^2)/k}$$

where k is the pressure divided by the density and supposed constant.

3. (a) Using Green's theorem, find the expression for kinetic energy T of a liquid. 4

(b) A velocity field is given by

$$\vec{q} = \frac{(-y\hat{i} + x\hat{j})}{x^2 + y^2}$$

Determine whether the flow is irrotational or not. 4

Or

Show that in irrotational motion, the velocity cannot be a maximum in the interior of the fluid.

4. *(a)* Define pressure and hydrostatics paradox. 2

(b) State True or False : 1

When two fluids of different densities at rest under gravity do not mix, their surface of separation is a horizontal plane.

(c) Prove that densities at two points in a fluid at rest under gravity and in the same horizontal plane are equal. 3

(d) Find the pressure at a point in the lower layer of two given heavy homogeneous liquids which do not mix. 5

(e) In a uniform circular tube, two liquids are placed so as to subtend 90° each at the centre. If the diameter joining the two free surfaces be inclined at 60° to the vertical, then prove that the densities of the two liquids are as $\frac{\sqrt{3}+1}{\sqrt{3}-1}$. 6

Or

A small uniform tube is bent into the form of a circle whose plane is vertical. Equal quantities of two fluids of densities ρ and σ fill half the tube. Show that the radius passing through the common surface makes with the vertical an angle θ given by $\tan\theta = \frac{\rho-\sigma}{\rho+\sigma}$.

5. *(a)* Define force of buoyancy. 1

(b) A square lamina *ABCD*, which is immersed in water, has the side *AB* in the surface. Draw a line *BE* to a point *E* in *CD* such that the pressure on the two portions into which it divides the lamina may be equal. Find the ratio in which the line *BE* divides *CD*. 4

(c) Prove that the position of the centre of pressure of a plane area is independent of the inclination of the area to its vertical. 6

Or

Find the centre of pressure of a triangular area immersed in a liquid with its vertex in the surface and base horizontal.

(d) A closed cylindrical vessel with hemispherical ends is filled with water and placed with its axis horizontal. Find the resultant thrust on each of the ends and determine its line of action. 5

Or

A solid circular cone of uniform material and height h and of vertical angle 2α, floats in water with its axis vertical and vertex downwards and a length h' of the axis is immersed. The cone is bisected by a vertical plane through the axis and the two parts are hinged together at the vertex. Show that the two parts will remain in contact if $h' > h\sin^2\alpha$.

6. (a) Write the condition of equilibrium if a body floats in two liquids which do not mix. 2

(b) Define stable and unstable equilibrium. 2

(c) What are the different types of equilibrium in respect of metacentre with respect to the positions of the centre of gravity? 3

(d) A rod of small cross-section and of density ρ has a small portion of metal of weight $\frac{1}{n}$th that of the rod attached to one extremity. Prove that the rod will float at any inclination in a liquid of density σ, if $(n+1)^2\rho = n^2\sigma$. 5

Or

If W_1, W_2, W_3 be the apparent weights of a given body in fluids whose specific gravities are S_1, S_2, S_3, then prove that

$$W_1(S_2 - S_3) + W_2(S_3 - S_1) + W_3(S_1 - S_2) = 0$$

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