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6 SEM TDC MTMH (CBCS) C 13

2025

(May)

MATHEMATICS

(Core)

Paper : C–13

(Metric Space and Complex Analysis)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* Write the symmetric property of metric space. 1

(b) Write when a subspace Y of a metric space will be completed. 1

(c) Write when a metric is called a trivial metric. 2

(d) Define an open set in a metric space. 2

(e) Write when a metric space is called complete. 2

(f) Answer any *two* questions from the following : 6×2=12

(i) If (X, d) be a metric space and $x, y, z \in X$ be any three distinct points, then prove that

$$d(x, y) \geq |d(x,z) - d(z, y)|$$

(ii) Prove that in a metric space (X, d) each closed sphere is a closed set.

(iii) Prove that interior of a set is an open set.

2. (a) Write when a mapping from one metric space into another is said to be continuous. 1

(b) Define a contracting mapping. 2

(c) Define uniform continuity in a metric space. 2

(d) Show that every contraction mapping is continuous. 4

(e) Let X and Y be metric spaces and f be a mapping of X into Y. If f is a constant mapping, show that f is continuous. 6

Or

Prove that a subspace of the real line R is connected if and only if it is an interval.

3. (a) Write when a function of a complex variable is called a many-valued function. 1

(b) Write the area of a parallelogram having sides z_1 and z_2. 1

(c) Write the nature of the singularity of the function

$$f(z) = \frac{\sin z}{z}$$ 1

(d) Show that $|z_1 z_2| = |z_1||z_2|$. 2

(e) Find the points where the function

$$f(z) = \frac{z}{z^2+1}$$

is not continuous. 2

(f) Show that $\sin^2 z + \cos^2 z = 1$. 3

(g) Prove the necessary condition for a function to be analytic. 5

Or

Find the image of the semi-infinite strip $x \geq 0$, $0 \leq y \leq \pi$ under the transformation $w = e^z$.

4. (a) $e^z = 0$, for some complex number z. State true or false. 1

(b) Show that $e^{2+3\pi i} = -e^2$. 2

(c) Define a simply connected domain. 2

(d) Evaluate $\int_0^{\pi/6} e^{i2t}\,dt$. 3

(e) Evaluate $\int_C \frac{z+2}{z}\,dz$, where C is the semi-circle $z = 2e^{i\theta}$ $(0 \leq \theta \leq \pi)$. 4

Or

Find $\text{Im}\, f(z)$, where

$$\text{Re}\, f(z) = e^x(x\cos y - y\sin y)$$

of an analytic function $f(z)$.

5. (a) Write when the sequence $\{z_n\}$ converges. 1

(b) If a series of complex numbers converges, then write to which nth term converges. 1

(c) Find the limit of the sequence defined by $z_n = -2 + i\frac{(-1)^n}{n^2}$, $n = 1, 2, 3, \cdots$ 3

(d) Expand $f(z) = \log(1+z)$ in a Taylor series about $z = 0$. 5

Or

Prove that the series $\sum_{n=1}^{n-1} \frac{z^{n-1}}{2^n}$ converges for $|z| < 2$.

6. (a) Write the statement of Laurent's theorem. 2

(b) Find the Laurent series for

$$f(z) = \frac{z}{(z-1)(z-3)}$$

when $0 < |z-1| < 2$. 6

Or

Investigate the uniform convergence of the series $\sum_{n=0}^{\infty} (-1)^n (z^n + z^{n+1})$.

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